

An Abstract Approach to Consequence Relations II

In the late 80s Blok and Pigozzi introduced the notion of an algebraizable logic, intended to isolate propositional logics which are intrinsically the same as equational consequences relative to quasi-varieties of algebras. Later on, Blok and Jónsson developed a general theory of equivalence between structural consequence relations, which abstracts the kind of equivalence observed between algebraizable logics and relative equational consequences to arbitrary structural consequence relations on sets equipped with monoid actions. Blok-Jónsson approach was subsequently extended by Galatos, Tsinakis and others to more general categorical settings.

We generalize Blok-Jónsson theory of equivalence between structural consequence relation in such a way as to naturally accommodate multiset-based consequence relations as well. While Blok and Jónsson admit, in place of sheer formulas, a wider range of syntactic units to be manipulated in deductions (including sequents and equations), these objects are invariably aggregated via set theoretical union. Our approach is more general in that non-idempotent forms of premiss and conclusion aggregation, including multiset sum and fuzzy set union, are considered. We investigate multiset-based consequence relations using categorical methods, and provide analogues of the main results obtained in the general theory of structural consequence relations.