Remarks on the Exponential Rules in Linear Logic

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February 27, 2018

This talk is about:

- An observation on exponentials
- A related conjecture
- A proof of a special case of the conjecture

The Logical Framework

AIL = Intuitionistic Linear Logic with Weakening

$A \multimap B$

$$\frac{\Gamma, A \Rightarrow B}{\Gamma \Rightarrow A \multimap B}\ (\multimap R) \qquad \frac{\Gamma_1 \Rightarrow A \qquad \Gamma_2, B \Rightarrow C}{\Gamma_1, \Gamma_2, A \multimap B \Rightarrow C}\ (\multimap L)$$

$A \vee B$

$$\frac{\Gamma \Rightarrow A_i}{\Gamma \Rightarrow A_1 \vee A_2}\ (\vee Ri)_{i=1,2} \qquad \frac{\Gamma, A \Rightarrow C \qquad \Gamma, B \Rightarrow C}{\Gamma, A \vee B \Rightarrow C}\ (\vee L)$$

$A \wedge B$

$$\frac{\Gamma \Rightarrow A \qquad \Gamma \Rightarrow B}{\Gamma \Rightarrow A \wedge B}\ (\wedge R) \qquad \frac{\Gamma, A_i \Rightarrow B}{\Gamma, A_1 \wedge A_2 \Rightarrow B}\ (\wedge L)_{i=1,2}$$

$A \otimes B$

$$\frac{\Gamma_1 \Rightarrow A \qquad \Gamma_2 \Rightarrow B}{\Gamma_1, \Gamma_2 \Rightarrow A \otimes B}\ (\otimes R) \qquad \frac{\Gamma, A, B \Rightarrow C}{\Gamma, A \otimes B \Rightarrow C}\ (\otimes L)$$

The exponential

$$!A \quad \frac{\Gamma, !A, !A \Rightarrow B}{\Gamma, !A \Rightarrow B}\,(!c) \quad \frac{\Gamma, A \Rightarrow B}{\Gamma, !A \Rightarrow B}\,(!dr) \quad \frac{!\Gamma \Rightarrow A}{!\Gamma \Rightarrow !A}\,(!pr)$$

Formulas as resources

$A \wedge B$ One resource: Either A or B
by my choice: LHS
by your choice: RHS

$A \otimes B$ Two resources: Both A and B

$A \multimap B$ A one-time usable function
turning resources A into resources B

$!A$ arbitrarily often/unbounded A

$$\Gamma \Rightarrow A$$

From resources Γ, one can obtain (at least) the resource A

Question

Is $!A = A \otimes A \otimes A \otimes A \ldots$?

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$$A^{\otimes n} := \underbrace{A \otimes \ldots \otimes A}_{n \text{ times}} \qquad A^{(n)} := \underbrace{A, \ldots, A}_{n \text{ times}}$$

$$\frac{\Gamma_1 \Rightarrow A \qquad \ldots \qquad \Gamma_n \Rightarrow A}{\Gamma_1, \ldots, \Gamma_n \Rightarrow A^{\otimes n}}$$

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" $\Rightarrow$" **AIL** proves $!A \Rightarrow A^{\otimes n}$ for any n:

$$\dfrac{\dfrac{\dfrac{A \Rightarrow A \qquad \ldots \qquad A \Rightarrow A}{A^{(n)} \Rightarrow A^{\otimes n}}}{(!A)^{(n)} \Rightarrow A^{\otimes n}}\; n \text{ times } (!dr)}{!A \Rightarrow A^{\otimes n}}\; n \text{ times } (!c)$$

" $\Leftarrow$" fails. More precisely:

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There are Γ,A such that

$$\mathbf{AIL} \vdash \Gamma \Rightarrow A^{\otimes n} \text{ for all } n$$

but

$$\mathbf{AIL} \nvdash \Gamma \Rightarrow !A$$

The generous coffee machine

$$\textbf{AIL} \vdash D, !(D \multimap C \otimes D) \Rightarrow C^{\otimes n} \text{ for all n}$$

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$$\dfrac{\dfrac{D \Rightarrow D \qquad \dfrac{C, D, !(D \multimap C \otimes D) \Rightarrow C}{C \otimes D, !(D \multimap C \otimes D) \Rightarrow C}\,(\otimes L)}{\dfrac{D, D \multimap C \otimes D, !(D \multimap C \otimes D) \Rightarrow C}{D, !(D \multimap C \otimes D), !(D \multimap C \otimes D) \Rightarrow C}\,(dr)}\,(\multimap L)}{D, !(D \multimap C \otimes D) \Rightarrow C}\,(c)$$

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$$\mathbf{AIL} \vdash D, !(D \multimap C \otimes D) \Rightarrow C^{\otimes n} \text{ for all n}$$

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$$
\dfrac{\dfrac{C \Rightarrow C \qquad \ldots \qquad C \Rightarrow C}{C^{(n)} \Rightarrow C^{\otimes n}}}{C^{(n)}, D, !(D \multimap C \otimes D) \Rightarrow C^{\otimes n}} \text{(weak)}
$$

$$
\vdots\ (n-2) \times Q
$$

$$
C, C, D, !(D \multimap C \otimes D) \Rightarrow C^{\otimes n}
$$

$$
\vdots\ Q
$$

$$
C, D, !(D \multimap C \otimes D) \Rightarrow C^{\otimes n}
$$

$$
\vdots\ Q
$$

$$
D, !(D \multimap C \otimes D) \Rightarrow C^{\otimes n}
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But:

$$\textbf{AIL} \nvdash D, !(D \multimap C \otimes D) \Rightarrow !C$$

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$$\frac{!\Gamma \Rightarrow A}{!\Gamma \Rightarrow !A}\ (!\mathrm{pr})$$

- What can we say about (Γ, A) such that

$$\mathbf{AIL} \vdash \Gamma \Rightarrow A^{\otimes n} \quad \text{for all } n$$

and the corresponding sequences of proofs

$$P_n \vdash \Gamma \Rightarrow A^{\otimes n} \quad ?$$

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- Trivial case (1): $\Gamma = !\Gamma'$

$$\begin{array}{c} Q \\ \vdots \\ !\Gamma' \Rightarrow A \end{array} \quad \rightsquigarrow \quad \dfrac{\dfrac{\begin{array}{c} Q \\ \vdots \\ !\Gamma' \Rightarrow A \end{array} \quad \dots \quad \begin{array}{c} Q \\ \vdots \\ !\Gamma' \Rightarrow A \end{array}}{(!\Gamma')^{(n)} \Rightarrow A^{\otimes n}}}{!\Gamma' \Rightarrow A^{\otimes n}}\ !c$$

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- Trivial case (2): $\Gamma = \emptyset$

$$\begin{array}{c} Q \\ \vdots \\ \Rightarrow A \end{array} \quad \rightsquigarrow \quad \dfrac{\begin{array}{c} Q \\ \vdots \\ \Rightarrow A \end{array} \quad \dots \quad \begin{array}{c} Q \\ \vdots \\ \Rightarrow A \end{array}}{\Rightarrow A^{\otimes n}}$$

- Similarily,

$$P_1 \vdash D, !(D \multimap C \otimes D) \Rightarrow C$$

has repeatable parts.

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has repeatable parts.

- But:

$$P_n \vdash C^{\otimes 20}, D, !(D \multimap C \otimes D) \Rightarrow C^{\otimes n}$$

does not necessarily have repeatable parts unless $n > 20$.

Conjecture

Let Γ be a multiset of formulas and A a formula. Let $d = maxdepth(\Gamma)$ and $n = |\Gamma| \cdot 2^d + 1$.
Then any proof

$$P \vdash \Gamma \Rightarrow A^{\otimes n}$$

has "enough repeatable parts" to generalize to a recursive sequence (P_k)

$$P_k \vdash \Gamma \Rightarrow A^{\otimes k} \quad \forall k \geq n$$

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(?)Corollary

If **AIL** $\vdash \Gamma \Rightarrow A^{\otimes n}$ for $|\Gamma| \cdot 2^d + 1$, then **AIL** $\vdash \Gamma \Rightarrow A^{\otimes k}$ for all k.

(?)Corollary

If there is a sequence (P_n) such that if $P_n \vdash \Gamma \Rightarrow A^{\otimes n}$ for all n, then there is a recursive such sequence.

Proposition

Let Γ be a multiset of formulas not containing $\multimap$ and A a formula. Let $d = maxdepth(\Gamma)$ and $n = |\Gamma| \cdot 2^d + 1$.
Then any proof

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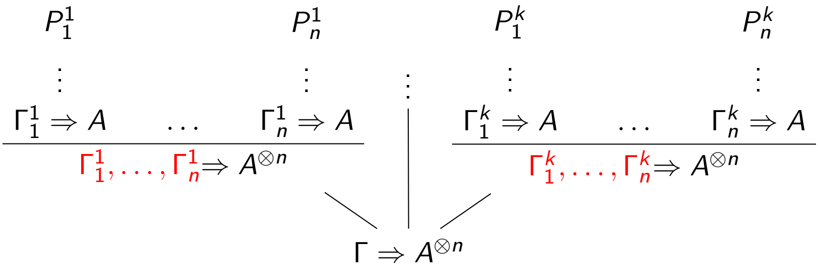
A normal form



- $\Gamma_1^i, \ldots, \Gamma_n^i$ splitting sequence
- only rule below splitting sequences: $(\wedge L)$,$(\vee L)$,$(\otimes L)$,(!c),(!dR) (no weakening)

Γ-grammar

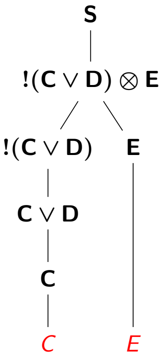
Every splitting sequence in P is generated by the following grammar:

$$N = \mathbf{S} \cup \{\mathbf{A} \mid A \in Subf(\Gamma)\}$$
$$T = Subf(\Gamma)$$

Production rules:

$$\mathbf{S} \mapsto \mathbf{A_1} \dots \mathbf{A_n} \qquad \text{where } \Gamma = A_1, \dots, A_n$$
$$\mathbf{A} \vee \mathbf{B} \mapsto \mathbf{A} \mid \mathbf{B}$$
$$\mathbf{A} \wedge \mathbf{B} \mapsto \mathbf{A} \mid \mathbf{B}$$
$$\mathbf{A} \otimes \mathbf{B} \mapsto \mathbf{A}\,\mathbf{B}$$
$$\mathbf{!A} \mapsto \mathbf{!A}\,\mathbf{!A} \mid \mathbf{A}$$
$$\mathbf{A} \mapsto A$$

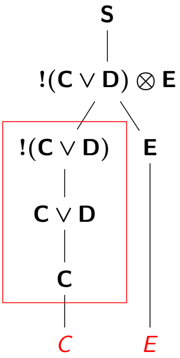
parsing tree



proof

$$\dfrac{\dfrac{\dfrac{C, E \Rightarrow A^{\otimes n} \qquad \begin{matrix} Q \\ \vdots \\ D, E \Rightarrow A^{\otimes n} \end{matrix}}{C \vee D, E \Rightarrow A^{\otimes n}}}{!(C \vee D), E \Rightarrow A^{\otimes n}}}{!(C \vee D) \otimes E \Rightarrow A^{\otimes n}}$$

parsing tree



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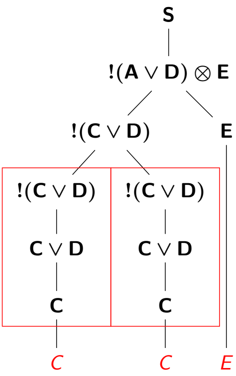

proof

$$\dfrac{\dfrac{\dfrac{\color{red}{C, E} \Rightarrow A^{\otimes n} \qquad \begin{array}{c} Q \\ \vdots \\ D, E \Rightarrow A^{\otimes n} \end{array}}{C \vee D, E \Rightarrow A^{\otimes n}}}{!(C \vee D), E \Rightarrow A^{\otimes n}}}{!(C \vee D) \otimes E \Rightarrow A^{\otimes n}}$$

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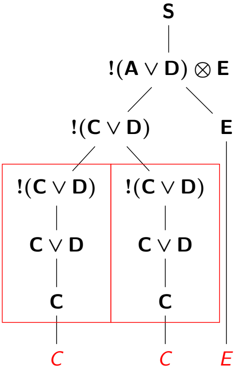

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proof

$$\dfrac{\dfrac{\dfrac{\dfrac{C, C, E \Rightarrow A^{\otimes n} \qquad \dfrac{\begin{array}{c} Q \\ \vdots \\ D, E \Rightarrow A^{\otimes n} \end{array}}{C, D, E \Rightarrow A^{\otimes n}}\,(\mathrm{w}) \qquad \dfrac{\begin{array}{c} Q \\ \vdots \\ D, E \Rightarrow A^{\otimes n} \end{array}}{D, D, E \Rightarrow A^{\otimes n}}\,(\mathrm{w})}{C \vee D, C \vee D, E \Rightarrow A^{\otimes n}}}{!(C \vee D), !(C \vee D), E \Rightarrow A^{\otimes n}}}{!(C \vee D), E \Rightarrow A^{\otimes n}}}{!(C \vee D) \otimes E \Rightarrow A^{\otimes n}}$$

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- The number of short ($\leq d$) paths in Γ-parsing trees is bound by

$$|\Gamma| \cdot 2^d$$

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- All formulas in $\Gamma^i_{k_i}$ can be duplicated!

The full picture

WLOG $\Gamma^i_{k_i} = \Gamma^i_1$

$$
\dfrac{\begin{array}{c} P^1_1 \\ \vdots \\ \Gamma^1_1 \Rightarrow A \end{array} \quad \ldots \quad \begin{array}{c} P^1_n \\ \vdots \\ \Gamma^1_n \Rightarrow A \end{array}}{\underline{\Gamma^1_1}, \ldots, \Gamma^1_n \Rightarrow A^{\otimes n}}
\qquad \vdots \qquad
\dfrac{\begin{array}{c} P^k_1 \\ \vdots \\ \Gamma^k_1 \Rightarrow A \end{array} \quad \ldots \quad \begin{array}{c} P^k_n \\ \vdots \\ \Gamma^k_n \Rightarrow A \end{array}}{\underline{\Gamma^k_1}, \ldots, \Gamma^k_n \Rightarrow A^{\otimes n}}
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$$
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$$\Gamma \Rightarrow A^{\otimes(n+1)}$$

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If yes, the infinitary rule

$$\frac{\Gamma \Rightarrow A \qquad \Gamma \Rightarrow A \otimes A \qquad \Gamma \Rightarrow A \otimes A \otimes A \qquad \dots}{\Gamma \Rightarrow \omega \cdot A}$$

is simulated by the finitary rule

$$\frac{\Gamma \Rightarrow A^{\otimes n}}{\Gamma \Rightarrow \omega \cdot A}$$

for large enough n.