A Logic for 'Maybe Because' Based on an Extension of Classical Logic with a Relevant Implication

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The aim of this paper is threefold. First, we generalize the relevant logic **PCR** from [3] in two different ways: predicative level and nesting of (relevant) implications. Next, we use the resulting logic, which we call **CLR**, as the base for the nonmonotonic logics $\mathbf{CLR^r_a}$ and $\mathbf{CLR^m_a}$. Both contain a connective that is most naturally interpreted as "maybe because". Finally, we show that both logics can be used to generate potential explanations on the basis of why-questions and that they have sensible applications in the context of abduction.

The logic **PCR** was designed by Diderik Batens for his introductory logic courses and first presented in a Dutch textbook ([2]), the current edition of which is still used by the first author. The textbook has always been accompanied by a set of computer programs (also designed by Diderik Batens) that allow students, among other things, to make proofs in Classical Propositional Logic (**PC**), in Classical Predicative logic (**CL**), and in **PCR**, and to obtain feedback on their solutions as well as hints when they are stuck, even for premises and conclusions of their choice. The computer programs (which by now have been used by several thousands of students) are currently being reimplemented as a web application by the second author.

The logic **PCR** is an extension of **PC**, with a simple relevant implication $\rightarrow$ for which nesting is not allowed. The extension is conservative (for all formulas A in the language of **PC**, $\vdash_{\mathbf{PC}} A$ iff $\vdash_{\mathbf{PCR}} A$), and a formula of the form $A \rightarrow B$ is a **PCR**-theorem iff it is a tautological entailment as defined in [1, §15]. Replacement of (Material) Equivalents does not hold in **PCR**, and as one will expect, the Deduction Theorem holds for the material implication, but not for the relevant one. The implication is relevant in the sense that no implication paradox holds for it, but the consequence relation is not relevant. So, for instance, $\nvdash_{\mathbf{PCR}} p \rightarrow (q \vee \neg q)$, but $p \vdash_{\mathbf{PCR}} q \vee \neg q$ and also $p \vdash_{\mathbf{PCR}} q \rightarrow q$.

In [3], a Fitch style characterization is presented for **PCR** as well as a possible world semantics and an algebraic semantics. We generalize both the proof system and the possible world semantics so that nested relevant implications can be handled and extend the result to the predicative level. We moreover give a modal characterization and present a nice embedding of **CLR** in **CL**.[1]

Having presented **CLR**, we extend its language with the fusion operator $\circ$ from relevant logic ($A \circ B =_{df} \neg(A \rightarrow \neg B)$) as well as a unary connective **?** and a binary

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[1] In the above mentioned computer programs, the decision procedure for **PCR** is based on the tableau method from [3]. In the new application, the decision procedure is based on the embedding in **CL** and proceeds in the same way as the one used for **CL**.

connective $\triangleleft$.[2] Where $A_1(\alpha), \ldots, A_n(\alpha)$ are atoms and $n \geq 1$, formulas of the form $(A_1(\alpha) \circ \ldots \circ A_n(\alpha)) \to B(\alpha)$ will be read as "$A_1(\alpha) \circ \ldots \circ A_n(\alpha)$ is a potential explanation for $B(\alpha)$", or "$B(\alpha)$, maybe because $A_1(\alpha) \circ \ldots \circ A_n(\alpha)$". A formula of the form $\mathbf{?}A$ will be read as "Why A?".

The resulting logic is called $\mathbf{CLR_a}$ and forms the base for the nonmonotonic logics $\mathbf{CLR_a^r}$ and $\mathbf{CLR_a^m}$. Both logics wil be characterized in the standard format of adaptive logics (see [4]). We present their (dynamic) proof theory as well as their semantics and show that their completeness is dependent only on the completeness of $\mathbf{PLR_a}$. (Once a nonmonotonic logic is characterized as an adaptive logic in standard format, the completeness proof is generic and depends only on the completeness of the so-called lower limit logic which is in our case $\mathbf{CLR_a}$.)

Unlike $\mathbf{CLR_a}$, $\mathbf{CLR_a^r}$ and $\mathbf{CLR_a^m}$ warrant that a formula of the form $(A_1(\alpha) \circ \ldots \circ A_n(\alpha)) \to B(\alpha)$ only follows from a set of premises if none of the *explanantia* is "irrelevant" for $B(\alpha)$. They morever enable one to derive (in a defeasible way) potential explanations as hypotheses.

We show that the logics $\mathbf{CLR_a^r}$ and $\mathbf{CLR_a^m}$ have interesting applications in the context of abduction and compare them to other formal logics for abduction (see, for instance, [8], [7], and [6]). We argue that the proposed logics offer an elegant solution to problems several other formal logics for abduction suffer from and which are related to the properties of "strengthening the antecedent" and "weakening the consequent". We also argue that they meet the criticism on the existing adaptive logics for abduction that can be found in [5].

We end by briefly mentioning some variants (for instance, in which a different relevant implication is used than the one from **PCR**). We conclude that for this kind of application one needs a logic that is as strong as **CL** but that contains a relevant implication as well.

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