

The classification of all the subvarieties of $\mathbb{D}\text{NMG}$

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Extended Abstract

Nilpotent minimum t -norm $*_{\text{NM}}$ [Fod95] was one of the first examples of a left-continuous but not continuous t -norm. The logic related to $*_{\text{NM}}$, NM , was introduced by Esteva and Godo in [EG01]. In the same paper they presented a generalisation of NM , the logic of Weak Nilpotent Minimum, WNM , and the related algebraic semantics, the variety of WNM -algebras WNM . WNM is an extension of MTL , the logic of all left continuous t -norms and their residua [EG01, JM02]. Several extensions of WNM have been extensively studied in the literature. In particular, Gödel logic G , Drastic Product DP ([ABV14], firstly introduced as S_3MTL in [Nog06]), Revised Drastic product RDP ([Wan07, BV12], based on the t -norm introduced in [Jen02]), NMG [WWP05], NM [EG01], and classical Boolean logic B . During the years a number of topics concerning WNM and its algebraic semantics has been investigated: the papers [EGN10, Mar16, NEG08, ABV17, GCNE05] are only few examples. WNM has been extensively studied in [Nog06], where the problem of axiomatising its extensions has been partially solved. The task of characterising and axiomatising the lattice of extensions of a given extension of WNM has been accomplished in some cases. Gispert [Gis03], solved the case for NM . The lattice of extensions of G is well known. The extensions of EMTL , which is the largest common fragment of G and DP (i.e. $\text{EMTL} = \text{G} \wedge \text{DP}$, in the lattice of axiomatic extensions of MTL), have been axiomatised in [Bia15].

This talk is based on the results of [ABV18]. We introduce the variety $\mathbb{D}\text{NMG}$, the algebraic semantics of DNMG , which is an extension of WNM which is a particularly tame single-chain complete common fragment of G , NM and DP (and hence, of all the aforementioned extensions of WNM). We prove standard completeness for DNMG . Generalising Gispert's result, we describe

the structure of the lattice of subvarieties of \mathbb{DNMG} , showing that each one of them is generated by finitely many chains. We further provide a uniform way to axiomatise each one of these subvarieties.

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